Extended Abstract

Superconducting Copper Pairs in AA Bilayer Graphene

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ABSTRACT

We use a multi-band extended Hubbard model to describe superconductivity in AA-bilayer graphene. In addition to the usual chirality of single-layer graphene that arises from the helicity of its massless quasiparticles, this system is described by an additional index called the "cone-index". This issue has been given very little attention in the literature, despite a few articles noting its prominent role in phenomena such as Klein tunneling and Andreev reflection. In this manuscript, it is shown that the cone-index is a consequence of mirror symmetry and is itself a new and distinct type of chirality from the helicity of massless quasiparticles, which we term “mirror-chirality”. Due to this mirror symmetry, the Hamiltonian of bilayer graphene is block-diagonalized by applying a unitary transformation. This process is mathematically analogous to the reduction of the Dirac equation in the 4-component Dirac representation to the chiral Weyl representation, which acts on left-handed and right-handed 2-component Weyl spinors. The method is extended to include the superconducting state. It is shown that four electrons in the upper and lower layers are coupled to form two kinds of superconducting Cooper pairs with an extra left/right-handed chiral character. Furthermore, inter-band Cooper pairing between quasiparticles with different mirror-chirality indices is forbidden. This naturally leads to a hidden double-gap superconductivity with new aspects.

Keywords: Superconductivity, AA bilayer graphene, Cooper pair, Prohibition rule, Weyl representation, chirality

1. INTRODUCTION

Superconductivity in low-carrier-density systems like twisted bilayer graphene [1-3] and multilayer graphene [4-6] has attracted significant attention, particularly following the discovery of correlated phases, including Mott insulation and magic-angle superconductivity [1,2]. While most research focuses on Bernal-stacked or twisted configurations, AA-stacked bilayer graphene remains relatively understudied [11-15], despite its structural simplicity and easier intercalation capability [11,12]. Remarkably, AA-bilayers exhibit unique quasiparticle behaviors, including a conserved "conical index" during Klein tunneling [16], which enables potential "conetronic" applications. Recent studies also report superconductivity in intercalated AA-bilayers [4,5,17], where Andreev reflection is strongly dependent on this conical degree of freedom [18].

This work investigates AA-bilayer superconductivity by exploring mirror symmetry, an approach that extends beyond the ultrarelativistic Dirac-point limit of previous studies [16,18]. By using a unitary transformation analogous to Dirac-Weyl reduction, we decompose the system into even/odd sectors, thereby revealing that the conical index actually represents a mirror-symmetry quantum number. The AA-bilayer behaves mathematically as two decoupled monolayer-like bands with opposite chiralities. In this system, mirror symmetry restricts

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Cooper pairing to intra-cone interactions (four-electron clusters with identical chirality) while prohibiting inter-cone pairing. This framework is consistent with Hubbard model predictions [19-21], and it not only explains observed superconducting gaps and nodal structures but also simplifies the experimental analysis of order parameters. Our results highlight the crucial role of mirror symmetry in the correlated phases of AA-stacked systems.

2. THEORETICAL FRAMEWORK AND METHODS

We employ an extended Hubbard model for AA-stacked bilayer graphene:

$H = -\sum t_{ij\sigma}(a^{\dagger}_{i\sigma}b_{j\sigma} + h.c.) + \sum U_i(ni\uparrow ni\downarrow) + \sum V_{\langle ij\rangle}\, ninj$

Key transformations:

1. Mirror-symmetry-based unitary operation:
   $|\psi\pm\rangle = (1/\sqrt{2})(|\psi_1\rangle \pm |\psi_2\rangle)$

2. Band decomposition into even/odd sectors:
   $H \rightarrow H^{+} + H^{-}$

3. Quasiparticle operators:
   $\gamma^{\pm}_k = (1/\sqrt{2})(c_{kA} \pm c_{kB})$

The superconducting order parameter Δ manifests as: $\Delta\pm = \langle\psi\pm\uparrow\psi\pm\downarrow\rangle$

Key results:

1. Energy dispersion:
   $E\pm(k) = \sqrt{(\varepsilon\pm(k)^2 + \Delta\pm^2)}$

2. Mirror symmetry constraint:
   $\Delta_{inter\text{-}cone} \equiv 0$

3. Gap equations:
   $\Delta+ = -g\sum k(\Delta+/2E+(k))\tanh(E+(k)/2kBT)$
   $\Delta- = -g\sum k(\Delta-/2E-(k))\tanh(E-(k)/2kBT)$

The model reveals:

- Two distinct superconducting gaps ($\Delta^{+} \neq \Delta^{-}$)
- Forbidden inter-cone pairing ($\Delta_{inter}=0$)
- Nodal structures determined by mirror symmetry

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3. CONCLUSION

This study investigates the superconducting properties in AA-stacked bilayer graphene. Using the principles of mirror symmetry and Dirac/Weyl representations, the research demonstrates that the band structure of this material splits into even and odd sectors, resulting in quasiparticles that possess a novel mirror chirality. Superconducting pairing is found to occur exclusively between quasiparticles with identical mirror chirality, while the same symmetry principle strictly forbids interactions between quasiparticles of opposite chiralities. This mechanism naturally explains the emergence of two distinct types of Cooper pairs, which are analogous to left-handed and right-handed particles. The findings of this study have broader implications and are extended to trilayer systems (AAA/ABA). The research suggests that these systems will exhibit unique superconducting and Klein tunneling behaviors when compared to monolayer graphene. The study further notes that breaking this mirror symmetry eliminates these observed effects, a phenomenon that could have potential experimental signatures in intercalated bilayers. Lastly, the work suggests that a future analysis using a four-electron Hamiltonian could reveal charged Cooper pairs, paving the way for exciting experimental prospects.

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