Extended Abstract

Shock wave structure in plasmas with regularized Kappa-distributed electrons

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ABSTRACT:

In this study, the structure of ion-acoustic shock waves in an unmagnetized plasma consisting of warm ions and regularized Kappa-distributed electrons has been investigated. Using the reductive perturbation method, the nonlinear Korteweg-de Vries-Burgers (K-dVB) equation is derived for the dynamics of ion-acoustic shock waves. Here, the dissipation generated by the ion kinematic viscosity is also considered. The analytical solution of the K-dVB equation is obtained by employing the tangent hyperbolic (Tanh) method. In addition, the analytical conditions for the propagation of oscillatory and monotonic shock structures are also discussed in detail. It is observed that when the viscosity of the plasma medium is fairly small, the oscillating shock waves will be able to propagate in the present plasma medium. Furthermore, it is seen that the regularized Kappa-distributed electrons can modify the structure of shock waves. In other words, both positive and negative amplitude shock waves can be propagated in the present plasma model. The dependence of shock structures on the parameters related to electron distribution (i.e., κ and α) and the kinematic viscosity of ions are also discussed. The results of this study may be useful for understanding nonlinear phenomena in space/astrophysics plasmas where regularized Kappa distributed electrons are assumed.

Keywords: Ion-acoustic shock waves, Regularized Kappa distribution, K-dV Burger equation, Dispersive effects.

1. INTRODUCTION

One of the most important electrostatic modes is the ion-acoustic waves (IAWs). These waves are caused by ions fluctuations and in the presence of background electrons that provide the role of restoring force [1]. The structure of such waves is strongly dependent on the distribution of electrons and ions. Different distributions have been introduced for plasma particles of which, one is the superthermal (kappa) distribution. This

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distribution is actually a generalization of the Maxwell-Boltzmann distribution function in the form of a capable sequence of speed and it was first introduced by the Vasyliunas [2]. Many researchers have studied the distribution of ion-acoustic waves in superthermal electron-ion (e-i) or electron-positron-ion (e-p-i) plasmas [3-12]. The classical Kappa distribution has been widely employed to describe superthermal particle populations in space and laboratory plasmas. However, it exhibits certain mathematical limitations: (i) divergence of higher-order velocity moments within specific parameter regimes, and (ii) unphysical behavior at extremely high energies. To address these issues, the regularized Kappa distribution (RKD) was proposed [13]. This modified form of the Kappa distribution is constructed to (i) ensure the finiteness of all velocity moments, (ii) allow a smooth and physically consistent transition to a Maxwellian distribution in the limit as $\kappa \to \infty$, and (iii) preserve key thermodynamic and physical properties of the system. As a result, the RKD offers a more robust and physically meaningful framework for modeling plasma populations exhibiting superthermal tails. Liu [14] was the first to investigate ion-acoustic (IA) solitons in a plasma composed of cold fluid ions and electrons following a regularized Kappa distribution. Using Sagdeev's potential method, Liu demonstrated that IA solitons with positive potential exist for $\kappa > 1.5$, while those with negative potential arise for $\kappa < 1.5$. Subsequently, Lu and Liu [15] studied ion-acoustic solitons in an unmagnetized plasma consisting of cold ions and regularized κ-distributed electrons using the Korteweg–de Vries (K-dV) model. Wei and Liu [16] focused on dust ion-acoustic (DIA) solitons in dusty plasmas containing regularized Kappa-distributed electrons, examining the effects of the spectral index κ, cutoff parameter α, and electron concentration on the properties of DIA solitons. The structure of electron-acoustic (EA) waves in plasmas comprised of two electron populations (cold and hot), along with stationary ions, was investigated in Ref. [17]. In this study, the hot electrons were assumed to follow a regularized Kappa distribution. To the best of our knowledge, a comprehensive study of shock waves in a plasma system with warm ion fluid and regularized κ-distributed electrons has not been conducted before. In this paper, we investigate the fundamental features of shock wave structures-including uniform or periodic shock waves- in an unmagnetized plasma composed of warm ion fluid and regularized κ-distributed electrons.

2. BASIC EQUATIONS AND K-dVB EQUATION

We consider a one-dimensional and unmagnetized e-i plasma, consisting of warm ions and kappa-distributed electrons, which are considered as a fast-neutralizing background. The basic normalized equations for the nonlinear dynamics of ion-acoustic shock waves in such a plasma medium are presented by

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} - -\frac{5}{3}\sigma n^{-\frac{1}{3}}\frac{\partial n}{\partial x} + \eta\frac{\partial^2 u}{\partial x^2}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n. \tag{3}$$

Where the normalized density of the electron is given as

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$$n_e = 1 + b_1\phi + b_2\phi^2 + \cdots, \tag{4}$$

with

$$b_1 = \kappa - \frac{1}{2} + \kappa\alpha^2\left[1 + \frac{3}{2}\frac{U\left(\frac{5}{2}, \frac{5}{2}-\kappa; \alpha^2\kappa\right)}{U\left(\frac{3}{2}, \frac{3}{2}-\kappa; \alpha^2\kappa\right)}\right], \tag{5}$$

$$b_2 = \kappa^2 - \frac{1}{4} + 2\alpha^2\kappa\left(\kappa - \frac{1}{2}\right)\left[1 + \frac{3}{2}\frac{U\left(\frac{5}{2}, \frac{5}{2}-\kappa; \alpha^2\kappa\right)}{U\left(\frac{3}{2}, \frac{3}{2}-\kappa; \alpha^2\kappa\right)}\right] + \kappa^2\alpha^4\left[1 + 3\frac{U\left(\frac{5}{2}, \frac{5}{2}-\kappa; \alpha^2\kappa\right)}{U\left(\frac{3}{2}, \frac{3}{2}-\kappa; \alpha^2\kappa\right)} + \frac{15}{4}\frac{U\left(\frac{7}{2}, \frac{7}{2}-\kappa; \alpha^2\kappa\right)}{U\left(\frac{3}{2}, \frac{3}{2}-\kappa; \alpha^2\kappa\right)}\right], \tag{6}$$

Additionally, $\mathrm{U}(a, b, c)$ refers to the Kummer U function [18], also known as the Tricomi function, which appears in the mathematical formulation of the RKD [13,14] :

$$\mathrm{U}(a, b, z) = \frac{1}{\Gamma(a)}\int_0^\infty t^{a-1}(1+t)^{b-a-1}e^{-zt}dt. \tag{7}$$

where $\sigma = T_i/T_e$and η is the kinematic viscosity. κ and α are the superthermality index and cutoff parameters measure the deviation from Maxwellian equilibrium.

In order to solve the nonlinear set of equations (1)-(4), we apply the reductive perturbation method [18] where $S = S^{(0)} + \varepsilon\, S^{(1)} + \varepsilon^2 S^{(2)} + \cdots$, $(S^{(0)} = \{1, 0, 0\})$ and coordinate $\xi = \varepsilon^{1/2}(x - Vt)$ and $\tau = \varepsilon^{3/2}t$,, where V is the velocity of solitary waves. Therefore, after doing necessary calculations, we derived the K-dVB equation as

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = C\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{8}$$

where $A = \frac{3V}{2\left(V^2-\frac{5}{3}\sigma\right)} - \frac{5\sigma}{18V\left(V^2-\frac{5}{3}\sigma\right)} - \frac{b_2\left(V^2-\frac{5}{3}\sigma\right)^2}{V}$, $B = \frac{\left(V^2-\frac{5}{3}\sigma\right)^2}{2V}$ and $C = \frac{\eta_0}{2}$.

3. RESULTS AND DISCUSSION

In this study, the properties of shock ion-acoustic waves in a plasma containing hot ions and non-Maxwellian electrons (with regularized Kappa distribution) are investigated. Using the reduced perturbation method, a nonlinear equation for the dynamics of shock ion-acoustic waves (K-dVB) is derived, and the stability of the traveling waves is then discussed analytically. However, according to the analyses, it is determined that if the viscosity of the plasma medium is less than a critical value, oscillatory shock waves will be able to propagate. It is also observed that the presence of electrons with a regularized Kappa distribution corrects the structure of the shock waves and enables the existence of shock waves with positive and negative amplitudes. In other words, it is shown that the parameters related to the distribution of electrons and the viscosity coefficient of ions play an important role in the propagation of shock ion-acoustic waves. Finally, the results of this research can be useful in understanding the properties of shock waves in space plasmas, where electrons follow a regularized non-Maxwellian Kappa distribution. Also, the effect of an external magnetic field, collisions between particles, and determining the properties of waves under conditions where the nonlinear coefficient becomes negligible can be raised as topics to be addressed in future studies.

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