Extended Abstract

One-loop Renormalization of a Lorentz-violating Scalar Field Theory with a Kink in 1+1 Dimensions

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Abstract

This article investigates the renormalization of a scalar field theory with Lorentz symmetry violation in two space-time dimensions. This theory is suitable for describing a kink as a nontrivial background of space. As is known, the standard theory of kink is generally renormalizable with only one mass counter-term in two space-time dimensions. Here we introduce a Lorentz violation term, which has non-removable effects. We show that, up to one-loop order, in addition to a modification of the mass counter-term, to have a renormalized theory, the theory needs a field strength renormalization counter-term. These modifications directly affect the radiative corrections of physical quantities.

Keywords: Scalar Field Theory, Violation of Lorentz Symmetry, Renormalization, Soliton, Kink

INTRODUCTION

Lorentz symmetry violation, at high energies, is a suitable candidate for examining low-energy signals in a unified quantum theory of gravity with other forces. In this article, our focus is on an "extended model" in 1+1 space-time dimensions that is suitable for describing one-dimensional solitons with Lorentz symmetry violation. Specifically, we will perform the renormalization of such a theory up to one-loop order. We can add the term $21\eta\mu\nu\partial\mu\phi\partial\nu\phi+21\delta\mu\nu\partial\mu\phi\partial\nu\phi$ to the scalar field Lagrangian. This term leads to Lorentz symmetry violation that cannot be resolved by a variable change or redefinition of coordinates or fields.

One-loop Renormalization of Scalar Field Theory

In current practice, if the space-time dimension is less than four, the scalar field theory is renormalizable, and particularly in two space-time dimensions, it is super-renormalizale. We can observe that for $\phi4$ theory, the superficial degree of divergence of the first one-loop diagram is D = -2, which is finite. Higher orders are also all finite. Therefore, this counter-term is not needed for renormalization in 1+1 dimensions. We also do not need the counter-term $\delta3\phi2$ in 1+1 dimensions. For renormalizing the standard $\phi4$ theory in two space-time dimensions, only one counter-term $\delta1m2\phi2$ is sufficient.

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Spontaneous Symmetry Breaking in Kink Theory

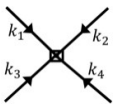
Spontaneous symmetry breaking occurs when the Lagrangian of a system has a specific symmetry, but the ground state of the system does not have that symmetry. The theory describing a one-dimensional soliton, called a kink, is one of the simplest examples for studying spontaneous symmetry breaking. For renormalizing this theory, the same single counter-term δ1m2ϕ2 is sufficient. Therefore, as Sidney Coleman [1] pointed out, renormalization in two dimensions is equivalent to normal ordering.

Lorentz Symmetry Violation

Violation in the time direction has no effect on the kink's properties. The symmetry violation term in the space direction takes the form $-\frac{1}{2}c\phi^2\left(u^\mu \partial_\mu \phi\right)^2$ and consequently we have:

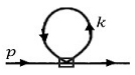
$$L(x)=\frac{1}{2}(\partial_\mu\phi)^2 - m_0^2\phi^2 - m_0\sqrt{\lambda}\phi^3 - \frac{\lambda_0}{4}\phi^4 - \frac{1}{2}c\phi^2\left(\partial_\mu\phi\right)^2$$

This Lagrangian adds the following vertex relative to the original Lagrangian:



$$=2ic(k_1\times k_2+k_1\times k_3+k_1\times k_4+k_2\times k_3+k_2\times k_4+k_3\times k_4)$$

This very term causes another diagram to be added in the expansion of the two-point function:



This new term is clearly infinite and has a direct effect on δZ and δm.

By applying the renormalization condition on the two-point function for δm, we have:

$$\delta m = -\frac{1}{8\pi}\int_{-\infty}^{+\infty}\frac{dk(\lambda+2ck^2)}{\sqrt{k^2+2m^2}}.$$

Also for δZ:

$$\delta Z=\frac{c}{4\pi}\int_{-\infty}^{+\infty}\frac{dk}{\sqrt{k^2+2m^2}}.$$

As a result, the counter-terms δm and δZ with Lorentz symmetry violation in two dimensions, up to one-loop order, are modified with the above values.

Conclusions

We were able to perform the renormalization process for this new theory. As a result of performing this process, we observed that in addition to modifying $\delta 1 m 2 \phi 2$, a new counter-term is also needed for field strength renormalization up to one-loop order. Therefore, with the existence of Lorentz symmetry violation, the modification of these counter-terms will have a direct impact on corrections higher than tree level for all physical quantities arising from Lorentz symmetry violation.

References

[1] S. Coleman, Quantum sine-Gordon equation as the massive Thirring model. Phys. Rev. D, 11 (1975) 2088, DOI :10.1103/PhysRevD.11.2088.