Extended Abstract

Coupling and Energy Transfer between Longitudinal and Radial Fields in a Parabolic Plasma Wakefield Accelerator Driven by a Proton Beam

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Abstract:

This study presents a theoretical model for accelerating charged particle beams in a plasma channel with a parabolic density profile. A Gaussian proton beam is used to excite longitudinal and radial wakefields, described by Whittaker functions. The radial wakefield exhibits anomalous behavior, with its amplitude increasing as a function of the distance from the beam axis. This phenomenon is attributed to a gradual energy transfer from on-axis longitudinal oscillations to radial oscillations near the channel wall. Two-dimensional simulations performed with the L-CODE show excellent agreement with the analytical results. Furthermore, a comparison of the wakefields at temperatures of 0 eV and 1 eV reveals a reduction in amplitude with increasing temperature, which is caused by enhanced ion mobility and intensified Landau damping—a wave-particle interaction mechanism occurring in collisionless plasmas.

Keywords: Gaussian Profile, Whittaker Function, Parabolic Plasma Channel, Wakefields, L-CODE.

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1. INTRODUCTION

Plasma wakefield acceleration has garnered significant attention in recent years [1, 2]. Most studies have focused on plasma channels with a parabolic density profile and minimum density on the axis, owing to the properties of plasma channeling. In contrast, for drivers composed of charged particles, a plasma with an inhomogeneous radial density profile that peaks on the axis is particularly important. In a previous study [3], an analytical method was presented for calculating the wakefields excited by a charged particle beam in a plasma with a parabolic density profile, and an exact solution to the governing equations was obtained using Whittaker functions. The results of that study revealed unusual behavior, where the radial electric field increased with distance from the beam. The current study mainly investigated and explained this anomalous characteristic. To validate the analytical results, two-dimensional simulations were conducted using the L-CODE, applying conditions and parameters consistent with the theoretical model [3]. The simulation results show strong agreement with the analytical predictions. Furthermore, the wakefields and their potential have been simulated and compared at two different plasma temperatures using the aforementioned code.

2. THEORITICAL MODEL

In the theoretical model presented in a study [3], a high-density beam is considered, which propagates through an axisymmetric plasma and displaces the plasma electrons, setting them in motion. The plasma has a density profile given by $n = n_0\left(1 - \frac{r^2}{r_{pl}^2}\right)$, where r_{pl} is the plasma radius and r is the radial coordinate. The peak plasma density n_0 is on the beam propagation axis along the z-direction. Figures 1(a) and 1(b) show the variations of the longitudinal and radial electric fields in three-dimensional space, respectively. As evident from the figures, the magnitude of the longitudinal electric field approaches zero with increasing radial distance (r) and decreases when moving away from the rear of the drive beam. In contrast, the radial electric field approaches zero on the central axis (r=0), but its amplitude increases with distance from the rear of the beam.

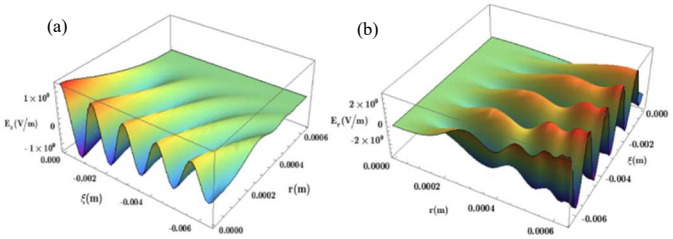


Figure 1. Behavior of (a) longitudinal electric field and (b) radial electric field in three dimensions as a function of r and ξ.

A comparison of Figures 1(a) and 1(b) indicates that the energy of the wakefield, which was initially stored in longitudinal oscillations, gradually transfers to radial oscillations in the channel's edge region. The total electric field, $E_{tot} = \sqrt{E_z^2 + E_r^2}$, shows that E_{tot} decreases with increasing distance from the rear of the beam, as illustrated in Figure 2.

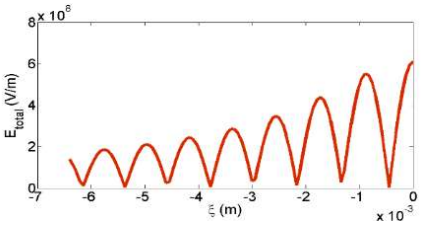


Figure 2. Total electric field from the longitudinal and radial field components.

3. SIMULATIONS

The simulations were performed using the two-dimensional relativistic electromagnetic code L-CODE [4]. A simulation window measuring 25 (in ξ) by 5 (in r) in units of (c/ω_p) was utilized, with step sizes of 0.05 for both r and ξ in the same units. The drive beam was modeled with 2×10^8 macro-particles per layer, while the plasma was represented by 20,000 macro-particles. Additional plasma parameters and the geometric dimensions of the drive beam, consistent with those employed in the analytical calculations, are listed in Table 1.

Table 1. Beam and plasma parameters for the simulation.

Parameters	Values
r_{pl}	$=2.97\ \frac{c}{\omega_{p0}} 0.64\times10^{-3}\text{m}$
N	1×10^{11}
σ_r	$0.43\times10^{-3}\ \text{m} = 2\frac{c}{\omega_{p0}}$
σ_z	$1\times10^{-4}\ \text{m} = 0.46\frac{c}{\omega_{p0}}$
k_{p0}	$4.65\times10^{3}\ \text{m}^{-1}$
n_0	$6\times10^{14}\ \text{cm}^{-3}$

Figures 3(a) and 3(b) show the behavior of the longitudinal electric field and radial electric field along the longitudinal direction, respectively. According to the simulation data, the amplitude of the longitudinal electric field decreases with distance from the rear of the beam, which is consistent with the analytical results. In contrast, the radial wakefield excited by the proton beam increases with distance from the rear of the beam.

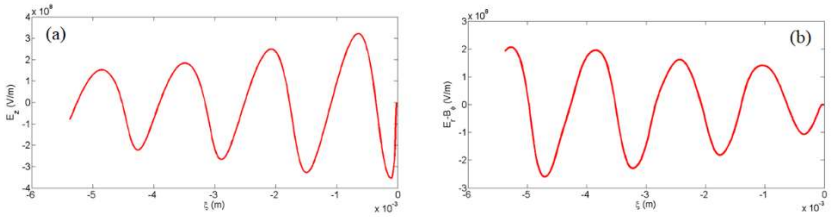


Figure 3. The behavior of (a) the longitudinal and (b) radial wakefield along the longitudinal direction on the axis.

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In Figures 4a and 4b, the longitudinal and radial wakefields are compared at temperatures of zero and 1 eV. When the plasma electrons have a non-zero initial temperature, their trajectories become slightly deflected compared to the zero-temperature case, causing some of these electrons to reach the axis earlier than those in the zero-temperature scenario. The difference between the two temperatures becomes more apparent at larger distances from the rear of the beam [2]. For the radial wakefield, an increase in the temperature of the plasma electrons reduces the amplitude compared to the zero-temperature case. This amplitude reduction becomes more pronounced at larger distances from the beam.

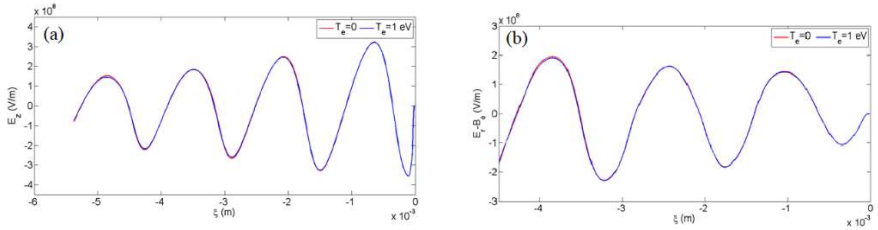


Figure 4. The behavior of (a) the longitudinal and (b) radial wakefield along the longitudinal direction on the axis at temperatures of zero and 1 eV.

4. CONCLUSION

A notable finding from this modeling of the parabolic density profile is the distinct behavior of the field components: the amplitude of the radial electric field excited by the beam increases with distance from its rear, while the amplitude of the longitudinal wakefield decreases under the same conditions. However, the rate of increase in the radial field amplitude is lower than the rate of decrease in the longitudinal field amplitude, ultimately leading to a reduction in the total effective field potential as the distance from the beam increases. These observations can be explained by the gradual transfer of energy stored in the initial longitudinal oscillations to radial oscillations in the channel edge region. To validate the analytical results, simulations were performed using the two-dimensional L-CODE, applying parameters consistent with the theoretical model. The simulation results showed strong agreement with the analytical predictions. In these simulations, the variations of the longitudinal and radial electric fields were carefully examined along both the beam propagation direction and the radial direction. Additionally, the behavior of the longitudinal and radial wakefields was compared at two different plasma temperatures. When plasma electrons have a non-zero initial temperature, a slight deviation in their trajectories is observed compared to the zero-temperature case, causing some electrons to reach the axis earlier than in the zero-temperature scenario. This temperature-induced effect becomes more pronounced at larger distances from the beam.

5. REFERENCES

[1] E. Esarey, C.B. Schroeder, W.P. Leemans, Physics of laser-driven plasma-based electron accelerators, Rev. Mod. Phys. 81 (2009) 1229–1285. https://doi.org/10.1103/RevModPhys.81.1229.
[2] C. Joshi, E. Adli, W. An, C.E. Clayton, S. Corde, S. Gessner, M.J. Hogan, M. Litos, W. Lu, K.A. Marsh, W.B. Mori, N. Vafaei-Najafabadi, B. O'Shea, X. Xu, G. White, V. Yakimenko, Plasma wakefield acceleration experiments at FACET II, Plasma Phys. Control. Fusion. 60 (2018) 034001. https://doi.org/10.1088/1361-6587/aaa2e3.
[3] Y. Golian, M. Aslaninejad, D. Dorranian, Whittaker functions in beam driven plasma wakefield acceleration for a plasma with a parabolic density profile, Phys. Plasmas. 23 (2016)013109. https://doi.org/10.1063/1.4940347.
[4] LCODE, LCODE - plasma wakefield simulation code. https://lcode.info, 2019–2025 (accessed 1 July 2026).